Mid-Cities Math Circle $(MC)^2$
Games
September 10, 2026

Warm-up Problems

Problem 1. Initially there are 20 checkers on the table. Two players take turns removing 1, 2, 3, or 4 checkers. The player who removes the last checker wins. Who has a winning strategy?

Problem 2. There are 2026 empty squares in a row. Player A and Player B take turns, starting with Player A. On each turn, a player writes either 1 or -1 in an empty square. Once all squares are filled, the numbers are multiplied together. Player A wins if the product is 1, and Player B wins if it is -1. Who has a winning strategy? What if there are 2027 squares?

Problem 3. A pile contains 2026 stones. On each turn, a player removes either 1 stone, 2 stones, or a positive multiple of 3 stones, without taking more than remain. Whoever takes the last stone wins. Determine the winning player and a strategy. What changes with 2027 stones?

More Difficult Problems

Problem 4. On a table there are 20 coins. Two players take turns removing exactly 2, 5, or 6 coins, provided that enough coins remain. The first player who cannot make a move loses. Who has a winning strategy?

Problem 5. There is a row of 10 coins on the table, each with a positive integer value visible to both players. On each turn a player takes one of the two end coins. After all coins have been taken, the player with the higher total value wins; equal totals give a draw. Prove that the first player can guarantee a win or a draw. Can a strict win always be guaranteed? What about a row of 2026 coins?

Problem 6. A stone starts in the bottom left square of a board with 2026 rows and 2027 columns. Player A and Player B move alternately, starting with Player A. Each move takes the stone upward or rightward by a positive number of squares. The player who reaches the top right square wins. Who has a winning strategy? What if the board has 2027 rows and 2027 columns?

Problem 7. Fix positive integers a, b with $a + b = 2027$. Starting from a positive integer t, two players alternately subtract either a or b from the current number. Whoever first obtains a negative number loses. Find infinitely many values of t for which the first player can win for every such choice of a, b.

Problem 8. Player A first writes the numbers $1, 2, \ldots, 2026$ in any order. Thereafter Player B and Player A take turns. On each turn, a player either rearranges the numbers in the current sequence or deletes one of them. The sequence must remain nonempty, and no sequence may be repeated during the game. The player who cannot make a legal move loses. Find a winning strategy for one of the players.

Problem 9. Alex and Katy use an initially empty 8×8 board. Alex moves first and marks one empty square with an A. Katy then marks two empty squares sharing an edge with a K in each. They continue alternating these moves until one player cannot move. Katy's score is the total number of squares marked K. What is the largest score she can guarantee, regardless of Alex's choices?

Problem 10. Player A and Player B alternately choose unused integers from $1, 2, \ldots, 2027$, with Player A starting. After each choice, add all the numbers chosen so far by both players. The player making the choice loses if this sum cannot be expressed as the difference of the squares of two nonnegative integers. Determine who has a winning strategy.

Problem 11. Let $N = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$. Player A and Player B alternately write distinct composite divisors of N other than N itself. Among the written numbers, no pair may be relatively prime, and neither number in a pair may divide the other. Player A starts; whoever cannot move loses. Determine the winning player.

Problem 12. Two players use an infinite square grid. Player A moves first and marks an empty square with an O; Player B then marks an empty square with an X. They alternate in this way. A player wins upon marking five consecutive squares in a horizontal row or a vertical column. Diagonals do not count. Prove that Player B can prevent Player A from winning.

Problem 13. Discs occupy distinct squares of a 5×9 board. In each move, every disc must move to a square sharing an edge with its current square. Each disc must alternate horizontal and vertical moves, although different discs may choose different directions. All discs stay on the board and occupy distinct squares after every move. Prove that with 33 discs the moves must eventually stop, but that 32 discs can be placed so that the moves continue forever.

Problem 14. Alice and Bob alternately fill empty squares of a 6×6 board with rational numbers, starting with Alice. All 36 numbers must be distinct. When the board is full, the square containing the largest number in each row is colored black. Alice wins if the black squares contain a path from the top row to the bottom row, where consecutive squares may share an edge or a vertex. Otherwise Bob wins. Find a winning strategy for one player.