Mid-Cities Math Circle $(MC)^2$
Symmetric Polynomials
March 4, 2026

Warm-up Problems

Problem 1. If x, y, and z are positive numbers such that

$$xy = 24, \qquad xz = 48, \qquad yz = 72,$$

find $x + y + z$.

Problem 2. A rectangular box has total surface area 94 and the sum of the lengths of all 12 edges is 48. Find the sum of the lengths of its 4 interior diagonals.

Problem 3. Suppose that real numbers x, y, and z satisfy

$$x + y + z = 6, \qquad xy + yz + zx = 11, \qquad xyz = 6.$$

Find

$$x^3 + y^3 + z^3.$$

More Difficult Problems

Problem 4. Suppose that x, y, and z are three positive numbers that satisfy

$$xyz = 1, \qquad x + \frac{1}{z} = 5, \qquad y + \frac{1}{x} = 29.$$

Find

$$z + \frac{1}{y}.$$

Problem 5. Compute $\sqrt[3]{5 + 2\sqrt{13}} + \sqrt[3]{5 - 2\sqrt{13}}$.

Problem 6. Let a, b, c be the three roots of $p(x) = x^3 + x^2 - 333x - 1001$. Find $a^3 + b^3 + c^3$.

Problem 7. The equation

$$\left(x - \sqrt[3]{13}\right)\left(x - \sqrt[3]{53}\right)\left(x - \sqrt[3]{103}\right) = \frac{1}{3}$$

has three distinct real solutions r, s, and t for x. Calculate the value of $r^3 + s^3 + t^3$.

Problem 8. If $-1 < x < 1$ and $-1 < y < 1$, define the "relativistic sum" $x \oplus y$ to be

$$x \oplus y = \frac{x + y}{1 + xy}\,.$$

The operation $\oplus$ is commutative and associative. Let v be the number

$$v = \frac{\sqrt[7]{17} - 1}{\sqrt[7]{17} + 1}\,.$$

What is the value of

$$v \oplus v \oplus v \oplus v \oplus v \oplus v \oplus v \oplus v \oplus v \oplus v \oplus v \oplus v \oplus v\,?$$

(In this expression, $\oplus$ appears 13 times.)

Problem 9. If x, y, z are real numbers satisfying relations

$$x + y + z = 1 \quad \text{and} \quad \arctan x + \arctan y + \arctan z = \frac{\pi}{4},$$

prove that $x^{2n+1} + y^{2n+1} + z^{2n+1} = 1$ holds for all positive integers n.

Problem 10. Solve the system

$$\begin{cases} x + y + z = 1 \\ x^5 + y^5 + z^5 = 1 \end{cases}$$

in real numbers.

Problem 11. Let $p(x)$ be the polynomial $(1-x)^a(1-x^2)^b(1-x^3)^c \cdots (1-x^{32})^k$, where $a, b, \ldots, k$ are integers. When expanded in powers of x, the coefficient of x^1 is -2 and the coefficients of x^2, x^3, $\ldots$, x^{32} are all zero. Find k.

Problem 12. Show that set of real numbers x which satisfy the inequality

$$\sum_{k=1}^{70} \frac{k}{x - k} \geq \frac{5}{4}$$

is a union of disjoint intervals, the sum of whose lengths is 1988.