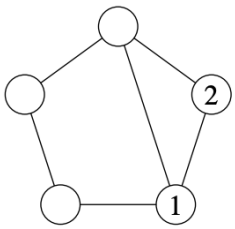
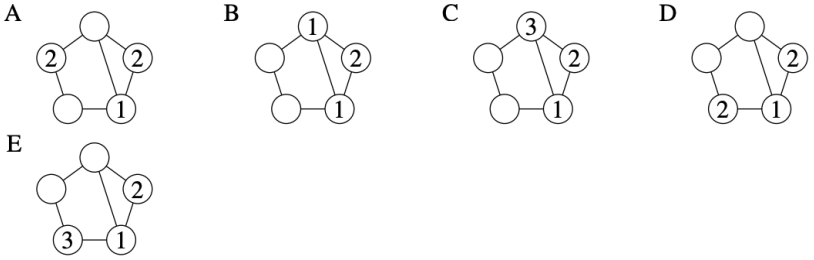
Mid-Cities Math Circle $(MC)^2$
Geometry and Games
November 12, 2025

Easier Problems

Problem 1. Two players X and Y take alternate turns in a game, starting with the diagram below.



At each turn, one player writes one of 1, 2 or 3 in an empty circle, so that no two circles connected by an edge contain the same number. A player loses when they cannot go. In each of the five diagrams below it is Y's turn. In which of the diagrams can Y's move ensure that X loses the game?



What about the case when both start from five empty circles?

Problem 2. 2024 points are marked on a circle. Two players alternately draw chords with endpoints among the marked points so that no two chords intersect in their interiors. The player who cannot move loses. Which player can force a win?

Problem 3. On a 4×4 board, players alternately place 1×2 (or 2×1) dominoes without overlap until no move remains. The player who cannot move loses. Who wins?

Problem 4. (Triangle Game) Anna and Bertha play the *Triangle Game* on a regular 2025-gon. The game consists of drawing a diagonal of the 2025-gon that has not been drawn previously and does not intersect any previously drawn diagonal in an inner point. The players take alternate moves starting with Anna. The game ends when no additional allowed diagonal can be drawn. If one or two triangles are created when the diagonal is drawn, the player labels the resulting triangle(s) with her initial. The player with the most labeled triangles at the end of the game wins. If they have the same number of labeled triangles, the game ends in a tie. Does Anna have a winning strategy, does Bertha have a winning strategy, or must the game end in a tie if both players play in an optimal way?

More Difficult Problems

Problem 5. Two players alternately place a token on an empty lattice point of a 4×6 rectangular grid of dots. A player X wins if, after the move of X, the board contains the four vertices of an axis-parallel rectangle whose four corner dots are occupied. Under optimal play, which player has a winning strategy?

Problem 6. On a convex n-gon ($n \geq 5$), two players alternately draw non-intersecting diagonals. A player *loses* if, after their move, there exists a quadrilateral whose two diagonals have not been drawn. Determine the winner in terms of n.

Problem 7. Initially $n \geq 3$ points lie on a circle. Players alternately draw *triangles* using three unused points as vertices, with the restriction that no edges may cross previously drawn edges. The first player unable to move loses. Determine the winner in terms of n.

Problem 8. (Quadrilateral Game) Same board and rules for drawing non-intersecting diagonals as in Problem 4, but a mover now labels any created *empty quadrilateral(s)* (with no diagonals inside). The player with more labeled quadrilaterals at the end wins (ties possible). With optimal play on a regular 2025-gon, what is the outcome?

Problem 9. Two players alternately fill a 5×5 board with 1's (first) and 0's (second), one per move. After the board is full, consider the nine 3×3 sub-squares; the first player's score is the *maximum* of these nine sums. What is the largest score the first player can guarantee?

Problem 10. Along a circle are 100 white points. An integer k is given, where $2 \leq k \leq 50$. In each move, we choose a block of k adjacent points such that the first and the last are white, and we paint both of them black. For which values of k is it possible for us to paint all 100 points black after 50 moves?