

Mid-Cities Math Circle (MC)²
Diophantine Equations
October 15, 2025

Easier Problems

Problem 1. How many different combinations of 5 bills and 2 bills can be used to make a total of 17? (Order does not matter.)

Problem 2. Soda is sold in packs of 6, 12 and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda?

Problem 3. How many ordered pairs of positive integers (m, n) satisfy $20m + 12n = 2012$?

More Difficult Problems

Problem 4. How many integers from 1 to 1000 inclusive can be written as the difference of squares of two nonnegative integers?

Problem 5. Let x, y be positive integers with

$$xy + x + y = 71 \quad \text{and} \quad x^2y + xy^2 = 880.$$

Find $x^2 + y^2$.

Problem 6. Determine all non-negative integral solutions $(n_1, n_2, \dots, n_{14})$ if any, apart from permutations, of the Diophantine Equation $n_1^4 + n_2^4 + \dots + n_{14}^4 = 1599$.

Problem 7. Find all solutions in positive integers m, n of

$$2^m - 3^n = 1.$$

Problem 8. Find all solutions in positive integers m, n of

$$3^n - 2^m = 1.$$

Problem 9. Find all solutions in positive integers x, y of

$$2^x + 1 = y^2.$$

Problem 10. There exist unique positive integers x and y that satisfy the equation $x^2 + 84x + 2008 = y^2$. Find $x + y$.

Problem 11. Find all positive integer solutions (x, y) of

$$y^2 - x(x+1)(x+2)(x+3) = 1.$$

Problem 12. Find all integer solutions (x, y) of

$$y^2 = x^3 + (x+4)^2.$$

Problem 13. Find all ordered pairs of integers (a, b) such that $a^2 + 4b$ and $b^2 + 4a$ are both perfect squares.