Mid-Cities Math Circle $(MC)^2$
Recursion and Sequences on AMC/AIME
September 10, 2025

Easier Problems

Problem 1. Buzz Bunny is hopping up and down a set of stairs, one step at a time. In how many ways can Buzz Bunny start on the ground, make a sequence of 6 hops, and end up back on the ground? (For example, one sequence of hops is up-up-down-down-up-down.)

Problem 2.
Let $a_1 = 2$, $a_2 = 3$, and $a_n = \dfrac{a_{n-1}}{a_{n-2}}$ for $n \geq 3$. Find a_{2025}.

Problem 3. Define a function on the positive integers recursively by $f(1) = 2$, $f(n) = f(n-1) + 1$ if n is even, and $f(n) = f(n-2) + 2$ if n is odd and greater than 1. What is $f(2025)$? And what about $f(2026)$?

More Difficult Problems

Problem 4. Everyday at school, Jo climbs a flight of 6 stairs. Jo can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

Problem 5. Let $a_1 = 1$, $a_2 = \frac{3}{7}$, and for $n \geq 3$ define

$$a_n = \frac{a_{n-2}\, a_{n-1}}{2a_{n-2} - a_{n-1}}.$$

Find a_{2025}.

Problem 6. Define $f(1) = f(2) = 1$ and for $n \geq 3$ set $f(n) = f(n-1) - f(n-2) + n$. Find $f(2025)$.

Problem 7. Define a sequence recursively by $F_0 = 0$, $F_1 = 1$, and $F_n =$ the remainder when $F_{n-1} + F_{n-2}$ is divided by 3, for all $n \geq 2$. Thus the sequence starts $0, 1, 1, 2, 0, 2, \ldots$ What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

Problem 8. For each positive integer n, let $S(n)$ be the number of sequences of length n consisting solely of the letters A and B, with no more than three As in a row and no more than three Bs in a row. What is the remainder when $S(2015)$ is divided by 12?

Problem 9. A collection of 8 cubes consists of one cube with edge-length k for each integer $k, 1 \le k \le 8$. A tower is to be built using all 8 cubes according to the rules:

- Any cube may be the bottom cube in the tower.
- The cube immediately on top of a cube with edge-length k must have edge-length at most $k+2$.

Find the number of different towers than can be constructed.

Problem 10. A mail carrier delivers mail to the nineteen houses on the east side of Elm Street. The carrier notices that no two adjacent houses ever get mail on the same day, but that there are never more than two houses in a row that get no mail on the same day. How many different patterns of mail delivery are possible?

Problem 11. The sequence (a_n) is defined by $a_0 = 1$, $a_1 = \sqrt[19]{2}$, and $a_n = a_{n-1}a_{n-2}^2$ for $n \ge 2$. What is the smallest $k \ge 1$ such that $a_1 a_2 \cdots a_k$ is an integer?

Problem 12. Let $\{X_n\}$ and $\{Y_n\}$ denote two sequences of integers defined as follows:

$X_0 = 1,\ X_1 = 1,\ X_{n+1} = X_n + 2X_{n-1}\ (n = 1, 2, 3, \dots),$

$Y_0 = 1,\ Y_1 = 7,\ Y_{n+1} = 2Y_n + 3Y_{n-1}\ (n = 1, 2, 3, \dots).$

Thus, the first few terms of the sequences are:

$X : 1, 1, 3, 5, 11, 21, \dots, \qquad Y : 1, 7, 17, 55, 161, 487, \dots.$

Prove that, except for the "1", there is no term which occurs in both sequences.