Mid-Cities Math Circle $(MC)^2$
Games
January 29, 2025

Warm-up Problems

Problem 1. Greta Grasshopper sits on a long line of lily pads in a pond. From any lily pad, Greta can jump 5 pads to the right or 3 pads to the left. What is the fewest number of jumps Greta must make to reach the lily pad located 2023 pads to the right of her starting position?

Problem 2. Initially there are 2025 checkers on the table. Two players take turns removing 1, 2, 3, 4, or 5 checkers. The winner is the one who removes the last checker. Who has a winning strategy?

Problem 3. Initially there is a 6×6 grid. Two players take turn each placing a domino (1×2 or 2×1 rectangle) on the grid. Dominoes cannot overlap, and the first player who cannot place a domino loses. Who can win the game? What if they start with a 2025×2026 grid?

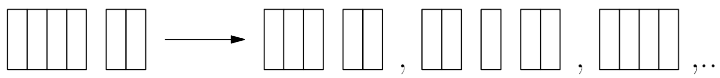
More Difficult Problems

Problem 4. Two players A and B play the following game. First A says 1, 2 or 3. Then B can add 1, 2 or 3 to the number the first player said. The game continues with the players playing alternately, in each turn adding 1, 2 or 3 to the previous number. For example, A can say 2, then B can say 5, then A could say 6, and so on. The player who says 2025 wins. Who has a winning strategy?

Problem 5. Initially there is a row of 10 coins on the table, and each coin has a positive integer value. Two players alternate turns. On each turn a player must take one of the two coins on either end of the row of remaining coins, so after each turn the row gets shorter by one. After all the coins have been taken, the player with the higher (or equal to the other) total value is the winner. Who can win the game? How about 2024 coins?

Problem 6. In a parlor game, the magician asks one of the participants to think of a three digit number (abc) where a, b, and c represent digits in base 10 in the order indicated. The magician then asks this person to form the numbers (acb), (bca), (bac), (cab), and (cba), to add these five numbers, and to reveal their sum, N. If told the value of N, the magician can identify the original number, (abc). Play the role of the magician and determine (abc) if $N = 3194$.

Problem 7. Arjun and Beth play a game in which they take turns removing one brick or two adjacent bricks from one "wall" among a set of several walls of bricks, with gaps possibly creating new walls. The walls are one brick tall. For example, a set of walls of sizes 4 and 2 can be changed into any of the following by one move: $(3, 2), (2, 1, 2), (4), (4, 1), (2, 2)$, or $(1, 1, 2)$:



Arjun plays first, and the player who removes the last brick wins. Does Beth have a strategy that guarantees a win if the starting configuration is $(6, 2, 1)$?

Problem 8. Nine tiles are numbered $1, 2, 3, \cdots, 9$, respectively. Each of three players randomly selects and keeps three of the tiles, and sums those three values. Find the probability that all three players obtain an odd sum.

Problem 9. On a 2025×2025 rectangular chessboard there is a stone in the lower leftmost square. Players A and B move the stone alternately, starting with A. In each step one can move the stone upward or rightward any number of squares. The player who moves it into the upper rightmost square wins. Who has a winning strategy? What if the players start with a 2024×2025 chessboard?

Problem 10. A solitaire game is played as follows. Six distinct pairs of matched tiles are placed in a bag. The player randomly draws tiles one at a time from the bag and retains them, except that matching tiles are put aside as soon as they appear in the player's hand. The game ends if the player ever holds three tiles, no two of which match; otherwise the drawing continues until the bag is empty. What is the probability that the bag will be emptied?

Problem 11. Alice and Bob play a game on a 6 by 6 grid. On his turn, a player chooses a rational number not yet appearing in the grid and writes it in an empty square of the grid. Alice goes first and then the players alternate. When all squares have numbers written in them, in each row, the square with the greatest number in that row is colored black. Alice wins if he can then draw a line from the top of the grid to the bottom of the grid that stays in black squares, and Bob wins if he can't. (If two squares share a vertex, Alice can draw a line from one to the other that stays in those two squares.) Find, with proof, a winning strategy for one of the players.