UT Arlington Mid-Cities Math Circle $(MC)^2$
Selected Problems in Geometry
April 26, 2023

Warm-up Problems

Problem 1. For which of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?

(a) a square

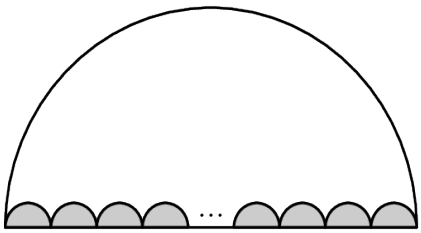
(b) a rectangle that is not a square

(c) a rhombus that is not a square

(d) a parallelogram that is not a rectangle or a rhombus

(e) an isosceles trapezoid that is not a parallelogram

Problem 2. In the figure below, N congruent semicircles lie on the diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let A be the combined area of the small semicircles and B be the area of the region inside the large semicircle but outside the semicircles. The ratio $A : B$ is $1 : 18$. What is N?

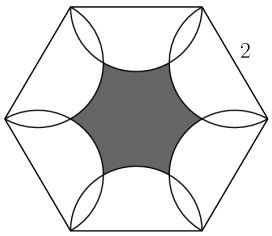


Problem 3. What is the largest number of solid 2-in $\times$ 2-in $\times$ 1-in blocks that can fit in a 3-in $\times$ 2-in $\times$ 3-in box?

More Difficult Problems

Problem 4. A child builds towers using identically shaped cubes of different colors. How many different towers with a height 8 cubes can the child build with 2 red cubes, 3 blue cubes, and 4 green cubes? (One cube will be left out.)

Problem 5. As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region – inside the hexagon but outside all of the semicircles?



Problem 6. All the numbers $2, 3, 4, 5, 6, 7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

Problem 7. Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?

Problem 8. All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

Problem 9. In $\triangle ABC$, $AB = 425$, $BC = 450$, and $AC = 510$. An interior point P is then drawn, and segments are drawn through P parallel to the sides of the triangle. If these three segments are of an equal length d, find d.

Problem 10. Let ABC be a triangle and point X lies on BC, so that and AX bisects angle A. Point Y lies on CA and BY bisects angle B. Angle A is $60°$. $AB + BX = AY + YB$. Find all possible values for angle B.