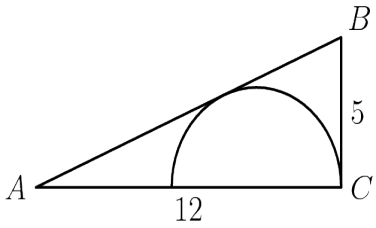
UT Arlington Mid-Cities Math Circle $(MC)^2$
Selected AMC Geometry Problems
November 30, 2022

Warm-up problems

Problem 1. What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?

Problem 2. In $\triangle ABC$, a point E is on $\overline{AB}$ with $AE = 1$ and $EB = 2$. Point D is on $\overline{AC}$ so that $\overline{DE} \parallel \overline{BC}$ and point F is on $\overline{BC}$ so that $\overline{EF} \parallel \overline{AC}$. What is the ratio of the area of $CDEF$ to the area of $\triangle ABC$?

Problem 3. In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



Problem 4. Rectangle $ABCD$ has sides $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

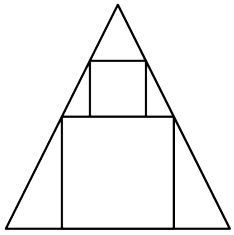
Harder problems

Problem 5. Four distinct points are arranged on a plane so that the segments connecting them have lengths a, a, a, a, $2a$, and b. What is the ratio of b to a?

Problem 6. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

Problem 7. A unit square is rotated $45°$ about its center. What is the area of the region swept out by the interior of the square?

Problem 8. A square with side length 3 is inscribed in an isosceles triangle with one side of the square along the base of the triangle. A square with side length 2 has two vertices on the other square and the other two on sides of the triangle, as shown. What is the area of the triangle?



Problem 9. In square $ABCD$, points P and Q lie on $\overline{AD}$ and $\overline{AB}$, respectively. Segments $\overline{BP}$ and $\overline{CQ}$ intersect at right angles at R, with $BR = 6$ and $PR = 7$. What is the area of the square?

Problem 10. Right triangle ABC has side lengths $BC = 6$, $AC = 8$, and $AB = 10$. A circle centered at O is tangent to line BC at B and passes through A. A circle centered at P is tangent to line AC at A and passes through B. What is OP?

Problem 11. Let $ABCD$ be a trapezoid with $AB||CD, AB = 11, BC = 5, CD = 19$, and $DA = 7$. Bisectors of $\angle A$ and $\angle D$ meet at P, and bisectors of $\angle B$ and $\angle C$ meet at Q. What is the area of hexagon $ABQCDP$?