UT Arlington Mid-Cities Math Circle $(MC)^2$
Selected AMC-AIME Geometry Problems
October 6, 2021

Warm-up problems

Problem 1. What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19?

Problem 2. Square $ABCD$ has side length 2. A semicircle with diameter AB is constructed inside the square, and the tangent to the semicircle from C intersects side AD at E. What is the length of CE?

Problem 3. Rectangle $ABCD$ has sides $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

Problem 4. Four distinct points are arranged on a plane so that the segments connecting them have lengths a, a, a, a, $2a$, and b. What is the ratio of b to a?

Harder problems

Problem 5. On square $ABCD$, point E lies on side AD and point F lies on side BC, so that $BE = EF = FD = 30$. Find the area of the square $ABCD$.

Problem 6. Circle C_1 has its center O lying on circle C_2. The two circles meet at X and Y. Point Z in the exterior of C_1 lies on circle C_2 and $XZ = 13$, $OZ = 11$, and $YZ = 7$. What is the radius of circle C_1?

Problem 7. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

Problem 8. A unit square is rotated 45° about its center. What is the area of the region swept out by the interior of the square?

Problem 9. Let $\triangle PQR$ be a triangle with $\angle P = 75^\circ$ and $\angle Q = 60^\circ$. A regular hexagon $ABCDEF$ with side length 1 is drawn inside $\triangle PQR$ so that side $\overline{AB}$ lies on $\overline{PQ}$, side $\overline{CD}$ lies on $\overline{QR}$, and one of the remaining vertices lies on $\overline{RP}$. Find the area of $\triangle PQR$.

Problem 10. Triangle ABC has $AB = 21$, $AC = 22$ and $BC = 20$. Points D and E are located on $\overline{AB}$ and $\overline{AC}$, respectively, such that $\overline{DE}$ is parallel to $\overline{BC}$ and contains the center of the inscribed circle of triangle ABC. Find the length of DE

Problem 11. In $\triangle ABC$, $AB = 425$, $BC = 450$, and $AC = 510$. An interior point P is then drawn, and segments are drawn through P parallel to the sides of the triangle. If these three segments are of an equal length d, find d.

Problem 12. Let $ABCD$ be a trapezoid with $AB||CD, AB = 11, BC = 5, CD = 19$, and $DA = 7$. Bisectors of $\angle A$ and $\angle D$ meet at P, and bisectors of $\angle B$ and $\angle C$ meet at Q. What is the area of hexagon $ABQCDP$?

Problem 13. Let $\overline{AB}$ be a diameter of circle ω. Extend $\overline{AB}$ through A to C. Point T lies on ω so that line CT is tangent to ω. Point P is the foot of the perpendicular from A to line CT. Suppose $\overline{AB} = 18$, and let m denote the maximum possible length of segment BP. Find m^2.

Problem 14. Triangle ABC has $BC = 20$. The incircle of the triangle evenly trisects the median AD. Find the area of the triangle.

Problem 15. In triangle ABC, $AB = 10$, $BC = 14$, and $CA = 16$. Let D be a point in the interior of $\overline{BC}$. Let points I_B and I_C denote the incenters of triangles ABD and ACD, respectively. The circumcircles of triangles BI_BD and CI_CD meet at distinct points P and D. Find the maximum possible area of $\triangle BPC$.