UT Arlington Mid-Cities Math Circle $(MC)^2$
Selected 2008 AMC10 Problems

Problem 1. (14/10B) Triangle OAB has $O = (0,0), B = (5,0)$, and A in the first quadrant. In addition, $\angle ABO = 90°$ and $\angle AOB = 30°$. Suppose that OA is rotated $90°$ counterclockwise about O. What are the coordinates of the image of A?

Problem 2. (16/10A) Points A and B lie on a circle centered at O, and $\angle AOB = 60°$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

Problem 3. (17/10A) An equilateral triangle has side length 6. What is the area of the region containing all points that are outside the triangle but not more than 3 units from a point of the triangle?

Problem 4. (19/10A) Rectangle $PQRS$ lies in a plane with $PQ = RS = 2$ and $QR = SP = 6$. The rectangle is rotated $90°$ clockwise about R, then rotated $90°$ clockwise about the point S moved to after the first rotation. What is the length of the path traveled by point P?

Problem 5. (20/10A) Trapezoid $ABCD$ has bases $\overline{AB}$ and $\overline{CD}$ and diagonals intersecting at K. Suppose that $AB = 9$, $DC = 12$, and the area of $\triangle AKD$ is 24. What is the area of trapezoid $ABCD$?

Problem 6. (24/10B) Quadrilateral $ABCD$ has $AB = BC = CD$, angle $ABC = 70$ and angle $BCD = 170$. What is the measure of angle BAD?

Problem 7. (11/10B) Suppose that (u_n) is a sequence of real numbers satisfying $u_{n+2} = 2u_{n+1} + u_n$, and that $u_3 = 9$ and $u_6 = 128$. What is u_5?

Problem 8. (13/10B) For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008th term of the sequence?

Problem 9. (9/10A) Suppose that

$$\frac{2x}{3} - \frac{x}{6}$$

is an integer. Which of the following statements must be true about x?
(A) It is negative.   (B) It is even, but not necessarily a multiple of 3.
(C) It is a multiple of 3, but not necessarily even.
(D) It is a multiple of 6, but not necessarily a multiple of 12.
(E) It is a multiple of 12.

Problem 10. (24/10A) Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

Problem 11. (15/10B) How many right triangles have integer leg lengths a and b and a hypotenuse of length $b + 1$, where $b < 100$?

Problem 12. (21/10B) Ten chairs are evenly spaced around a round table. Five married couples are to sit in the chairs with men and women alternating, and no one is to sit either next to or across from his/her spouse. How many seating arrangements are possible?

Problem 13. (23/10A) Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly two elements. In how many ways can this be done, assuming that the order in which the subsets are chosen does not matter?

More Challenging Problems

Problem 13: 2008 AIME I, Problem 2. Square $AIME$ has sides of length 10 units. Isosceles triangle GEM has base EM, and the area common to triangle GEM and square $AIME$ is 80 square units. Find the length of the altitude to EM in $\triangle GEM$.

Problem 14: 2008 AIME I, Problem 6. A triangular array of numbers has a first row consisting of the odd integers $1, 3, 5, \ldots, 99$ in increasing order. Each row below the first has one fewer entry than the row above it, and the bottom row has a single entry. Each entry in any row after the top row equals the sum of the two entries diagonally above it in the row immediately above it. How many entries in the array are multiples of 67?