UT Arlington Mid-Cities Math Circle $(MC)^2$
Polynomials

Problem 1. Find a polynomial with integer coefficients whose zeros include $\sqrt{2}+\sqrt{5}$.

Problem 2. Let $p(x)$ be a polynomial with integer coefficients. Assume that $p(a) = p(b) = p(c) = -1$, where a, b, c are three different integers. Prove that $p(x)$ has no integer zeros.

Problem 3. Let $P(x) = x^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ be a polynomial with integer coefficients. Suppose that there exist four distinct integers a, b, c, d with $P(a) = P(b) = P(c) = P(d) = 5$. Prove that there is no integer k with $P(k) = 8$.

Problem 4. (USAMO 1975) If $P(x)$ denotes a polynomial of degree n such that $P(k) = k/(k+1)$ for $k = 0, 1, 2, ..., n$, determine $P(n+1)$.

Problem 5. Find the remainder when you divide $x^{81} + x^{49} + x^{25} + x^9 + x$ by $x^3 - x$.

Problem 6. Find all polynomials $f(x)$ for which $xf(x-1) = (x+1)f(x)$.

Problem 7. Determine all polynomials $P(x)$ such that $P(0) = 0$ and $P(x^2+1) = P(x)^2 + 1$.