UT Arlington Mid-Cities Math Circle $(MC)^2$
Problem Solving Session II

Problem 1. Let $a_1 = 3$ and define $a_{n+1} = \frac{3a_n^2+1}{2} - a_n$ for $n \geq 1$. If n is a power of 3, prove that a_n is divisible by n.

Hint. Try to express a_n explicitly as a function of n.

Problem 2. Five married couples gather at a party. As they come in and greet each other, various people exchange handshakes but, of course, people never shake hands with themselves or with their own respective spouses. At the end of the party, one woman goes around asking people how many hands they shook, and she gets nine different answers. How many hands did she herself shake?

Problem 3. Let A_1, B_1, C_1 be the points on the sides BC, CA, AB (respectively) of the triangle ΔABC. Prove that the three circles circumscribed about the triangles ΔAB_1C_1, ΔBC_1A_1, and ΔCA_1B_1 intersect at one point.

Problem 4. A cube $3 \times 3 \times 3$ is made of cheese and consists of 27 small cubical cheese pieces arranged in the $3 \times 3 \times 3$ pattern. A mouse is eating the cheese in such a way that it starts at one of the corners and eats smaller pieces one by one. After he finishes one piece, he moves to the adjacent piece (pieces are adjacent if they share a face). Is it possible that the last piece mouse has eaten is the central one?

Problem 5. In how many different ways one can place 3 rooks on the cells of 9×2009 chessboard such that they do not attack each other?