UT Arlington Mid-Cities Math Circle $(MC)^2$
Problem Solving Session I

Problem 1. A king is placed in the left bottom corner of a 6 by 6 chessboard. At each step it can either move one square up, or one square to the right, or diagonally - one up and one to the right. How many ways are there for the king to reach the top right corner of the board?

Problem 2. Solve the equation

$$2\sqrt{1+x\sqrt{1+(x+1)\sqrt{1+(x+2)\sqrt{1+(x+3)(x+5)}}}} = x$$

Problem 3. If n is an integer, prove that the number $1+n+n^2+n^3+n^4$ is not divisible by 4.

Problem 4. Let $n > 3$ be an integer which is not divisible by 3. Two players A and B play the following game with $n \times n$ chocolate table. First, player A has to chose and remove one piece of the chocolate, without breaking other pieces. After his move, player B tries to partition the remaining chocolate into 3×1 (and 1×3) rectangles. If B manages to do so, then he/she is the winner. Otherwise the winner is A. Determine which player has a winning strategy and describe the strategy.

Problem 5. Let $ABCD$ be a rectangle. Let E be the end point of the perpendicular from A to BD. Let F be an arbitrary point of the diagonal BD between D and E. Let G be the intersection of the line CF with the perpendicular from B to AF. Let H be the intersection of the line BC with the perpendicular from G to BD. Prove that $\angle EGB = \angle EHB$.