UT Arlington Mid-Cities Math Circle $(MC)^2$
Inversion and Plane Geometry

Problem 1. Let $ABCD$ and $A_1B_1C_1D_1$ be two squares oriented in the same direction. Prove that AA_1, BB_1 and CC_1 are concurrent if $D = D_1$.

The *degree of point* A with respect to a circle k with center O and radius R is defined as $d_k(A) = OA^2 - R^2$. The *radical axis* of two circles $k_1(O_1, R_1)$ and $k_2(O_2, R_2)$ is the set of all points which have the same degree with respect to k_1 and k_2, i.e all A for which $d_{k_1}(A) = d_{k_2}(A)$.

Problem 2. Describe the radical axis of $k_1(O_1, R_1)$ and $k_2(O_2, R_2)$ if
(a) the circles are nonintersecting.
(b) the circles are intersecting.

Problem 3. Prove that the radical axes of three circles intersect in one point, provided their centers do not lie on a line.

Problem 4. Find the distance between the center P of the inscribed circle and the center O of the circumscribed circle of a triangle ΔABC in terms of the two radii r and R.

Problem 5. Prove that the altitude of ΔABC through C is the radical axis of the circles with diameters the medians AM and BN of ΔABC .