UT Arlington Mid-Cities Math Circle $(MC)^2$
Combinatorics

Part 1: Combinations

Problem 1. Helen has 15 skirts, 12 shirts and 3 pairs of shoes. In how many ways can Helen dress for tonight party (provided she likes her shoes matched)?

Problem 2. There are 10 houses on one side of a street, and you have three paints (green, yellow and red) to paint them.
   (a) How many ways are there to paint the houses?
   (b) The same question as in part (a) but with the addition that the city council requires that neighboring houses have different colors.

Problem 3. How many 5-letter "words" can one make out of 26 letters, under the condition that a word has at least one vowel?

Part 2: Permutations

Problem 4. Six horses participate in a race. How many ways are there to finish the race if we assume that no two horses finish simultaneously?

Problem 5. There are n boys and n girls in a dance class. How many ways are there to divide them in pairs for a tango?

Part 3: Binomial coefficients

Problem 6. A script-writer decided that in his plot, James Bond must have 10 girlfriends, Alice, Barbara, Clara, Denise, Eleanor, Fiona, Georgina, Hannah, Irene, and Janelle; exactly 3 of which are spies. How many different combinations of spies can there be?

Problem 7. How many ways are there to divide 12 identical oranges between 4 people?

Problem 8. The same question as in Problem 7, but everybody gets at least one orange.